1 Differential Equations

1.1 Differential Equations

The process is the following:

1. Separate Variables (multiply or divide to get the x and y's on opposite sides. dx and dy must always be on top).
2. Integrate both sides
3. Add $+C$ with the x side.

Example

Use integration to find the general solution to the differential equation

$$\frac{dy}{dx} = 2x(x-4)$$

We are finding $dy = 2x^2 - 8xdx$

Integrate both sides to get $y = \frac{2}{3}x^3 - \frac{8x^2}{2} + C$.

$y = \frac{2}{3}x^3 - 4x^2 + C$ is the general solution.

Exercise Find the particular solution of $f'(x) = 7x - 6$ knowing thta $f(1) = \frac{3}{2}$.

Exercise Use integration to find the general solution to the differential equation $\frac{dy}{dx} = \frac{x-1}{y-6}$

1.2 Euler's Method

This is basically baby steps with tangent lines.

The general procedure is

$$\text{New } y = \text{Old } y + \text{Slope(step size)}$$

Example

Consider a function whose slope is given by $\frac{dy}{dx} = 2xy$ with initial condition $f(1) = 1$.

Use Euler's method with a step size of 0.1 to approximate $f(1.3)$.

$f(1.1) \approx 1 + 2(1)(1)(.1) = 1.2$

$f(1.2) \approx 1.2 + 2(1.1)(1.2)(.1)$

Keep doing this to find $f(1.3)$.

1.3 Exponential Growth and Decay

Solving problems where the rate of growth is proportional to the amount present.

$$\frac{dy}{dt} = kt \implies \ln|y| = kt + C$$
$$\implies y = Ce^{kt}$$

Example

Write an equation for the amount Q of a radioactive substance with a half-life of 30 days, if 10 grams are present when $t = 0$.

$Q(t) = Ce^{kt}$.

$Q(t) = 10e^{kt}$, so we need to find k.

$5 = 10e^{30k}$, so $k = \frac{\ln\frac{1}{2}}{30}$

Exercise The balance in an account triples in 30 years. Assuming that interest is compounded continuously, what is the annual percentage rate?

Exercise In 1990 the population of a village was 21,000 and in 2000 it was 20,000. Assuming the population decreases continuously at a constant rate proportional to the existing population, estimate the population in the year 2020.

Exercise A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 500 present at a given time and 1,000 present 2 hours later, how many will there be 5 hours from the initial time given?