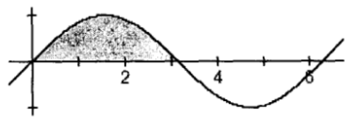
1 Calculus

Problems

1. Find the area of the shaded region (nearest square unit)



2. Which of the following sequences is divergent?

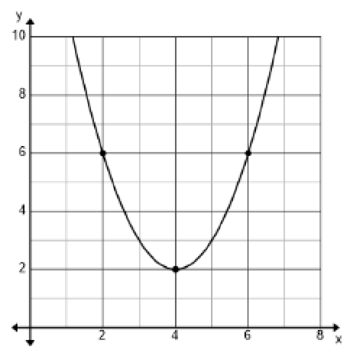
(A) $\left\{\frac{2n+1}{3n-2}\right\}$ **(B)** $\left\{\frac{-1^n}{n^2+n}\right\}$ **(C)** $\left\{\frac{(-1)^n(n+1)}{n+2}\right\}$ **(D)** $\left\{\frac{4n^2-n^3}{10+2n^3}\right\}$ **(E)** $\left\{\frac{6n^2+3n-1}{n^2+8n+16}\right\}$

3. If $f'(x) = 6x^2 - 4x + 1$ and $f(1) = 0$, find $f(-1)$.

4. $f(x) = 2x^3 - 6x + 1$ has an inflection point at:

5. Find the area (in square units) of the region bounded by $x = \frac{y^2+2}{2}$ and $x = y + 5$.

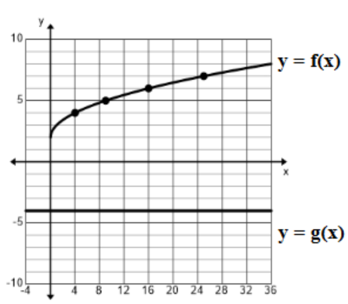
6. The graph of $f'(x)$ is shown below. If $f(1) = 2\frac{1}{3}$, then $f(2) =$ ______.



7. The point of inflection for the graph of $f(x)$ has coordinates (a, b). $a + b =$ ______. (nearest tenth)

x	-3	-2	-1	0	1	2
f(x)	10	-9	-10	-5	-6	-25

The following graph is used for problems 8 and 9.



8. Find the area between the curves $y = f(x)$ and $y = g(x)$ shown on the right over the interval $[4, 24]$. (nearest whole number)

9. Find the volume of the solid generated by revolving the region bounded by $y = f(x)$, the x-axis, the line $x = 4$ and the line $x = 24$ about the line $y = g(x)$. (nearest whole number)

10. Find the area of one petal of the rose curve $r = 6\cos(2\theta)$. (nearest tenth)

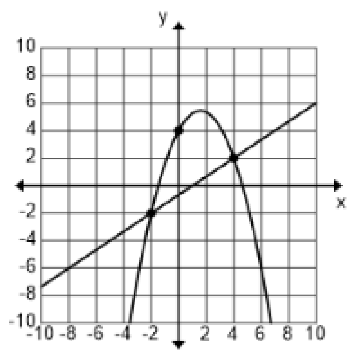
11. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{4^n}$.

12. Find the value of c in the open interval $(-8, 2)$ that satisfies the mean value theorem for the function $f(x) = \sqrt{6 - x}$. (nearest hundredth)

13. If you were going to evlauate $\int \frac{\cos x}{\sin^3 x} \mathrm{d}x$ using u-substitution, the best choice for u is ______.

14. If $f(x) = x^2 - 8x + 9$, then $\frac{f(x+h)-f(x)}{h} =$ ______.

15. Find the area bounded by thw two curves shown below. (nearest tenth)

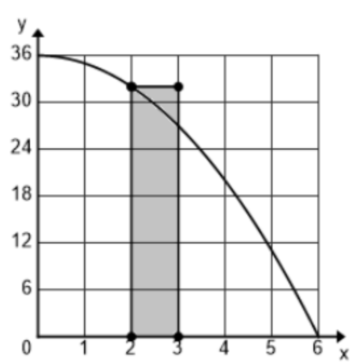


16. Consider the function $f(x) = \frac{1}{2}\cos(2x) + \frac{3}{2}\sin(x)$. Find the slope of the line tangent to the graph of $y = f(x)$ when $x = \pi$. (nearest tenth)

17. A balloon is rising straight up from a point on the ground 150 feet from a curious mouse. If the balloon is rising at a rate 8 ft/s, what is the rate of change of the angle of elevation of the balloon from the mouse when the balloon is 200 ft above the groud. (nearest hundredth)

18. A rectangular solid with a square base has a total surface area of 330 in^2. Find the maximum volume possible for such a solid. (nearest tenth)

Use the following graph for questions 19 and 20.



19. Find an approximation of the area bounded by the graph of $f(x) = 36 - x^2$ and the x-axis between $x = 1$ and $x = 5$. Use four rectangles of equal width and find the height of each rectangle using the left endpoint of the interval. One of the rectangles is shown above.

20. Find the exact area of the region bounded by the graph of $f(x) = 36 - x^2$ and the x-axis between $x = 1$ and $x = 5$. (nearest tenth)

21. Find the derivative of $F(x)$ if $F(x) = \int_0^{4x} \sin(t)\mathrm{d}t$.

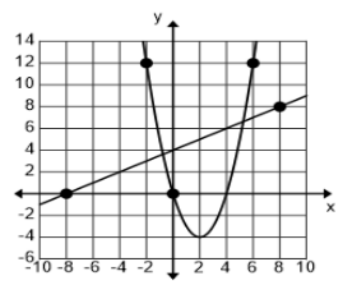
22. When evaluating $\int x^2 \cos(x)\mathrm{d}x$ using a u-substitution, the best choice for u is ______.

23. Let $f(x) = \sin(x)$ and let $P_5(x)$ be the fifth Maclaurin polynomial for $f(x) = \sin(x)$. Find the value of $\left|P_5\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{6}\right)\right|$. (nearest ten-millionth)

24. Find the length of the arc from $\theta = \frac{\pi}{6}$ to $\theta = \frac{\pi}{3}$ for the polar curve $r = 4 - 4\cos(\theta)$. (nearest tenth)

25. The point $A(6, b)$ lies on the graph of the parametric equations $x(t) = \sqrt{2t}$ and $y(t) = \frac{10}{t+2}$, $0 \le t \le 60$. $b =$ ______.

The following graph is used for problems 26 and 27.



26. The points of intersection of the curves shown on the right are P and Q. $PQ =$ ______. (nearest tenth)

27. Find the area bounded by the two curves shown on the right. (nearest tenth)

28. The ______Theorem states that "If f is a continuous, real valued function defined on an interval $[a, b]$, with $f(a) \neq f(b)$, and k is a real number between $f(a)$ and $f(b)$, then there exists some $c \in (a, b)$ such that $f(c) = k$."

29. Emmitt won the lottery and decided to purchase some land near Lubbock and raise emus. He wanted to build a pen in the shape of a rectangle to keep his emus. He has 480 feet of fencing to use for three sides of the pen. He will use one side of his large barn as the fourth side. What is the maximum area of the pen? (nearest square foot)

30. An 18-ft-long ladder rests against the wall of a building. The foot of the ladder begins to slide away from the building at a constant rate of 6 in/s. How fast is the top of the ladder sliding down the wall at the instant the ladder makes an angle of $30°$ with the wall? (nearest hundredth)

31. Find the area of the region in the first quadrant bounded by the graphs of $y_1 = 3 + \cos(x)$ and $y_2 = 2 - \cos(x)$ and the y-axis. (nearest tenth)

32. Find the slope of the line tangent to the curve $2y^2 - 6xy + 3x^3 - 4y = 8$ when $x = 1$ and $y > 0$. (nearest hundredth)

33. To evaluate $\int \sin^5(x)\cos(x)\mathrm{d}x$ using a u-substitution, the best choice for u is ______.

34. To evaluate $\int x^2 \sin(x)\mathrm{d}x$ using integration by parts, the best choice for u is ______.

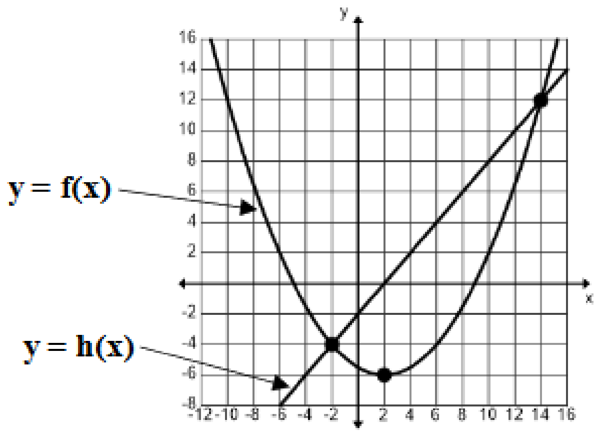
35. Experts from Texas Tech believe that Newberry State Park in Seminole is capable of supporting no more than 250 prairie dogs. On April 1, 2012, Anthony introduced the first pair of prairie dogs. On April 1, 2020 the population had reached 60 prairie dogs. Professor Cravens commissioned Carter to develop a logistic model of the prairie dog population. The logistic model predicts that there should be about ______prairie dogs in 2030.

For problems 36 and 37, consider the curve given by $x(t) = \sin(t)$ and $y(t) = t + \cos(t)$, $\frac{\pi}{2} \le t \le \frac{5\pi}{2}$. (rad)

36. Find the length of the curve. (nearest hundredth)

37. The tangent line when $t = \pi$ intersects the tangent line when $t = 2\pi$ at the point $P(a, b)$. $a + b =$ ______. (nearest hundredth)

For problems 38-41, consider the following graph.



38. The graphs of $y = f(x)$ and $y = h(x)$ intersect at the points P and Q. $PQ =$ ______. (nearest tenth)

39. The point $F(2, b)$ is the focal point of the parabola. $b =$ ______.

40. The area bounded by the graphs of $y = f(x)$ and $y = h(x)$ is ______. (nearest tenth)

41. If the area bounded by the graphs of $y = f(x)$ and $y = h(x)$ is revolved around the line $x = -6$, then the volume of the solid generated is ______. (nearest whole number)

42. Consider the graph of $h(x) = 2\ln(x) - \frac{1}{e^x}$. The slope of the line tangent to the graph of $h(x)$ at $x = 9$ is ______. (nearest thousandth)

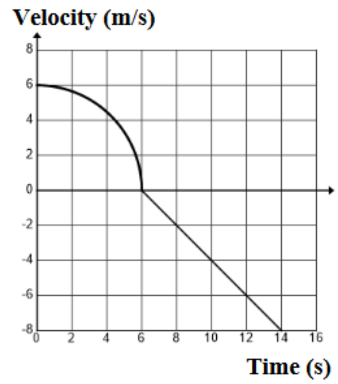
43. Consider the graph of $2xy^2 - 3y + 4x^2 = 13$. The y-intercept of the line tangent to the curve at the point where $y = 3$ and $x > 0$ is ______. (nearest tenth)

44. Farmer Fred wants to make a rectangular holding area for his dairy cattle using 640 feet of fence. He plans to use the back side of his barn as one of the sides. The maximum possible value of the holding area is ______square feet.

45. A 25-ft-long ladder rests against the wall of a building. The foot of the ladder begins to slide away from the building at a constant rate of 2 ft/s. How fast is the top of the ladder sliding down the wall at the instant the foot of the ladder is 7 feet from the wall? (nearest whole number)

46. The position of a particle is given by the parametric equations $x(t) = e^{.4t}$ and $y(t) = \ln(t^2 + 2)$ for $0 \leq t \leq 12$. Find the total distance traveled by the partial from $t = 2$ to $t = 10$. (nearest tenth)

Use the following graph for problems 47 and 48.



The graph consists of a quarter circle and a line segment. The graph represents the velocity of an object during a 14-second time interval.

47. Find the object's average velocity during the 14-second time interval $[0, 14]$. (Nearest hundredth)

48. Find the objects acceleration at $t = 10$ s.

Time	11:00 AM	12:00 PM	2:00 PM	6:00 PM	8:00 PM
People/minute	6	9	7	11	5

49. Suppose that Larry's Cafeteria in Millersview opens their doors at 11:00 AM and closes their doors at 8:00 PM. The table above shows the rate at which people entered the cafeteria, in people per minute, at various times on Saturday. Use a trapezoidal approximation with four subintervals to estimate the total number of people who dined at Larry's on Saturday.

50. The rate of change of a population of horned lizards at any time t, $t \geq 0$, is changing at a rate proportional to its population at time t. The population on March 1, 2000 was 180. On March, 1, 2004 the population was 210. What should the population be on March 1, 2033?

51. Consider the curve given by $f(x) = x^3 + 6x^2 - 4x + 2$. The local maximum of $f(x)$ is ______. (nearest tenth)

52. $\int 2^{-x} \mathrm{d}x =$ ______ $+ C$, where C is an arbitrary constant.

53. $\int \frac{13}{144+25\cos^2 x} \mathrm{d}x = \frac{1}{b} \tan^{-1}(\frac{b}{c} \tan x) + C$ where C is an arbitrary constant, $c > b > a > 0$ and a, b, c form a primitive Pythagorean triple. Find a.

54. The function $f(x) = x^3 - 3x^2 + 3$ has an inflection point at:

55. The area (in square units) of the region bounded by $y = 1 - x^2$ and the x-axis is:

56. Let f be a function such that it is continuous on $[a, b]$ and it is differentiable on (a, b). Then there exists at least one number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. This theorem is known as:

57. Let $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } 0 < x \end{cases}$ Which of the following statements is a false statement.

(A) f is continuous at 0 **(B)** the right hand derivative at 0 is 0
(C) the left hand derivative at 0 is 1 **(D)** f is not differentiable at 0 **(E)** $f(-1) = f(1)$

58. Find the area, in square units, of the figure bounded by $y = x^2 - x - 2$ and below the x-axis.

59. Let $f(x) = \frac{x-2}{3x+5}$. Find $f'(-1)$.

60. Find the digit in the ten-thousandths place of the series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$, when $x = \pi$.

61. A function $y = f(x)$ is continuous on $[a, b]$, if $f(a) < y_0 < f(b)$ then $y_0 = f(c)$ for some c in $[a, b]$. This theorem is the:

62. Let $f(x) = ax^5 - bx^4 - bx^3 + ax^2 + ax - b$. Find $f''(1)$.

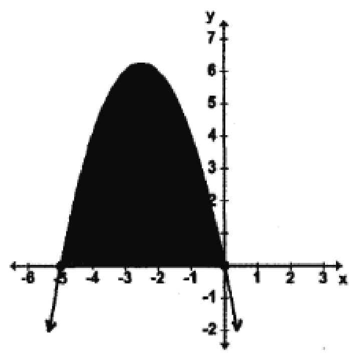
63. If $a_1 = -4, a_3 = -9$, and $a_4 = 13.5$ are terms of a geometric sequence, then $a_2 =$ ______.

64. How many points of intersection occur when $r = 2\cos\theta + 1$ and $\theta = \pi$ are graphed on a polar coordinate system?

65.

$$\sum_{k=0}^{2}(kx + (k+1)y) =$$

66. Find the area of the shaded region in square units.



67. If $f(x) = \frac{2x+3}{4x-5}$, then $f'(1) =$ ______

68. The slope of the line tangent to the curve $y = 2x^3 - 3x^2 - 5$ at $x = 2$ is 12. The point of intersection of the tangent line and curve is:

69. Evaluate $\int_{-n}^{n}(x^3 - 3x^2 - 5)\mathrm{d}x$

70. Find $f(2)$ when $f(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \ldots$. (nearest thousandth)

71. Let f be a function such that it is differentiable on (a, b) and continuous on $[a, b]$, and $f(a) = f(b) = 0$. Then there is a number c in (a, b) for which $f'(c) = 0$. This theorem is known as:

72. The function $f(x) = \frac{2}{x-1} + 18x$ is increasing at which of the following values of x?

73. Find the area (in square units) of the region bounded by $y = -x^2$ and $y = -4$.

74. Let $f(x) = \frac{4x+5}{3x}$. Find $f'(2)$.

75. Find an equation of the tangent line to the curve $y = \sqrt{9 - 4x}$ at the point $(-4, 5)$.

76. The point of inflection on the graph of $f(x) = 2x^3 - 6x^2 + 6x - 6$ is (a, b). Find b.

77. Find an equation of the line tangent to the curve $y = x^3 - 2x^2$ at the point $(1, -1)$.

78. The area (in square units) of the region bounded by $y = -x^2 - 4x$ and $y = 0$ is:

79. $\int(-x\sin x)\mathrm{d}x =$ ______ $+ C$, where C is some arbitrary constant.

80. If $f''(x) = 6$ and $f'(-1) = -8$ and $f(1) = 2$, then $f(-2) =$ ______ .

81. Find the instantaneous rate of change of the reciprocal of a number with respect to the number when the number is 4.

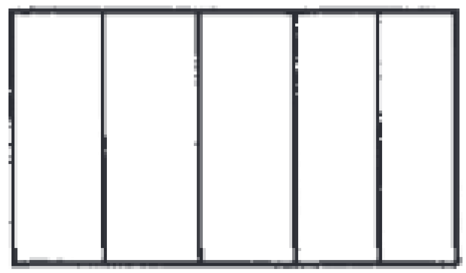
82. Let $f(x) = \frac{1}{x-1}$. Find the average rate of change of $f(x)$ over the interval $[2, 5]$.

83. Find the first term of the geometric sequence: $a, b, 44, c, 19\frac{5}{9}, \ldots$

84. If $f'(x) = 15x^2 - 6x + 2$ and $f(-1) = -9$, find $f(1)$.

85. $\int \sin(2x)\cos(2x)\mathrm{d}x =$ ______ $+ C$, where C is an arbitrary constant.

86. Elmoor Fudd is building a rectangular shaped pen for his porkie pigs. It will have 4 parallel fences dividing the pen into 5 sections as shown. If he has 600 feet of fencing, what is the maximum area of his pig pen?



87. Find the area of the region bounded by the graphs of $x = 4 - y^2$ and $x = 4 - 4y$.

88. If $f'(x) = 3x^2 - 5$ and $f(-1) = 4$, find $f(1)$.

89. Let $\frac{1}{x} + \frac{1}{y} = 1$. Find $D_x y$.

90. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2-2}{x-1}$

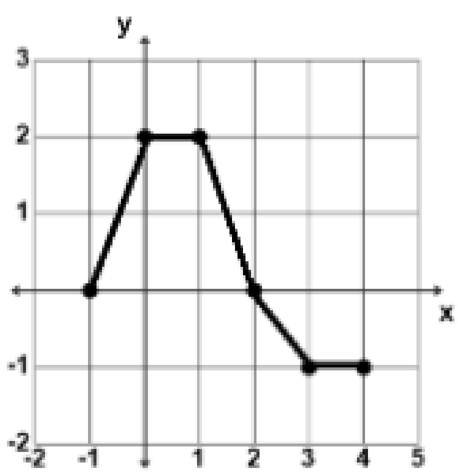
91. Let $f(x) = \begin{cases} 3+x & \text{if } x \le 1 \\ 3-x & \text{if } 1 < x \end{cases}$. Which of the following is/are true?

1. $\lim\limits_{x\to 1^+} f(x)$ exists 2. $\lim\limits_{x\to 1^-} f(x)$ exists 3. $f(x)$ is continuous

92. The function $f(x) = \begin{cases} nx^3 - x \text{ if } x \le 1 \\ mx^2 + 5 \text{ if } 1 < x \end{cases}$ is differentiable everywhere. Find n.

93. Let $f(x) = \frac{5x-2}{4+3x}$. Find $f'(-2)$.

94. Find the value of $\int_{-1}^{4} f(x)\mathrm{d}x$ for the piecewise-linear function, f, $-1 \le x \le 4$, shown below?



95. Mei Chado is 5' 4" tall. She is walking at a rate of 3 ft/sec toward a street light that is 16 feet tall. At what rate is the tip of her shadow moving? (nearest tenth)

96. Let $f(x) = 4x^2 - 4x + 1$. The tangent to $f(x)$ at (x, y) is parallel to $y = 4x - 2$. Find $x + y$.

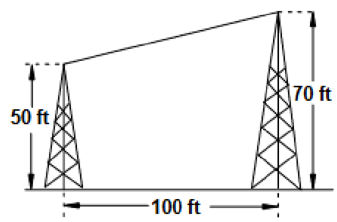
97. Let $f(x) = \begin{cases} -x+5 & x < -2 \\ x^2+1 & -2 \le x \text{ and } x \le 1 \\ 2x^3-1 & 1 \le x \end{cases}$. Which of the following is/are true?

(a) f is continuous at -2

(b) f is differentiable at $x = 1$

(c) f has a local minimum at $x = 0$

98. A cable is connected from the shorter tower to the taller tower. What is the minimum length of the cable? (nearest inch)



99. Find the area bounded by $f(x) = x^3, f(y) = -2$, and $f(y) = 1$. (square units)

100. What is the slope of the secant line to the graph of $f(x) = 2x^2 + 3x - 4$ passing through the points $(1, m)$ and $(-3, n)$?

101. Rusty Pipes has a leaky pipe dripping water onto the floor forming a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm? (nearest cm^2/min)

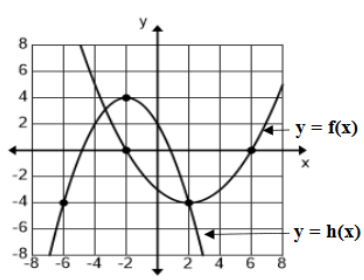
102. Let $f''(x) = 12x - 6, f'(0) = 4$, and $f(0) = -5$. Find $f(1)$.

103. Let $f(x) = |x - 5|$. How many of the following statements are always true?

a. $\lim_{x\to5^+} f(x)$ exists **b.** $\lim_{x\to5^-} f(x)$ exists **c.** $f(x)$ is continuous **d.** $f(x)$ is differentiable

104. The partial fraction decomposition of $\frac{x+8}{x^2+x-6}$ is $\frac{A}{x-2} + \frac{B}{x+3}$. $A + B =$ ______.

The following graph is used for problems 105, 106, 107, and 108.



105. The directrix of the graph of $y = h(x)$ is the line $y = c$. The value of c is ______.

106. If the parabolas intersect at points A and B, then $AB =$ ______. (nearest tenth)

107. The area of the region bounded by the graphs of the parabolas is ______. (nearest tenth)

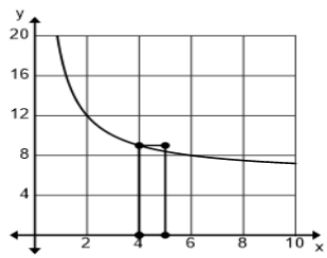
108. Find the arc length of the graph of $y = f(x)$ on the interval $[0, 8]$. (nearest tenth)

109. A rectangle is to be inscribed between the graph of $y = 16 - x^2$ and the x-axis with its base on the x-axis. What is the maximum area of such a rectangle? (nearest tenth)

110. Find the sum of the series. $2 - \frac{4}{3} + \frac{4}{15} - \frac{8}{315} + \ldots$

111. Find the area in the second quadrant bounded by the x-axis, the y-axis, and the graph of $r(\theta) = 2\theta + 3\sin(theta), 0 \le \theta \le 2\pi$. (nearest tenth)

The following graph is used for problems 112 and 113.



112. Consider the graph of $y_1 = \frac{12}{x} + 6$. Use the Left Rectangle Approximation Method with six rectangles of equal width to approximate the area bounded by the curves $y_1 = \frac{12}{x} + 6, y_2 = 0, x_1 = 2$, and $x_2 = 8$. One of the rectangles is shown above. (nearest hundredth)

113. Find the volume of the solid generated when the region bounded by the curves $y_1 = \frac{12}{x} + 6, y_2 = 0, x_1 = 2, x_2 = 8$ is revolved around the line $x = 12$. (nearest whole number)

114. The Panhandle Coffee shop keeps their dining area at a constant $72°$. They serve their famous coffee at exactly $175°$ and the temperature of the coffee changes at the rate $r(t) = -5.89e^{-.0766t}$ degrees per minute. Darius received a phone call at the moment his coffee was served and his coffee cooled for exactly five minutes before he was able to take his first sip. What was the temperature of the coffee when he took his first sip? (nearest whole number)

115. (rad) The derivative of the function f is given by $f'(x) = 2x^3 - 6\sin(x^2) + 2$. On the interval $(-2, 2)$, at which of the following values does f have a relative minimum? (nearest thousandth)

I. -0.535 **II.** 0.669 **III.** 1.260

For problems 116 and 117 use the following information:

(rad) The position of an object moving in the xy-plane is given by $(x(t), y(t)), 0 \le t \le \frac{5\pi}{12}$, with $\frac{\mathrm{d}x}{\mathrm{d}t} = 4t\sin(t)$ cm/s and $\frac{\mathrm{d}y}{\mathrm{d}t} = 4t\cos(t)$ cm/s. At $t = 0$, the position of the object is $(4, 8)$.

116. Find the speed of the object at $t = \frac{\pi}{6}$. (nearest hundredth)

117. The position of the object at $t = \frac{\pi}{3}$ is (a, b). $b =$ ______. (nearest hundredth)

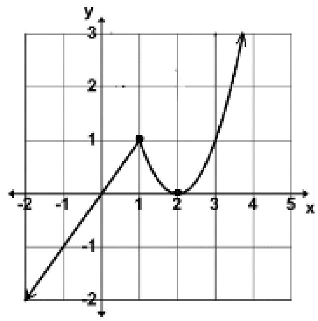
118. Consider the first quadrant region bounded by the y-axis, the line $x = 4$, the line $y = 10$, and the curve $y = 2\ln(5 - x)$. The region is the base of a solid by cross sections in which each cross section is a square perpendicular to the x-axis. What is the voluem of the solid? (nearest whole number)

119. The equation for the tangent line at $x = 3$ for the function $y = 2x^2 - 5$ is $y =$ ______.

120. Let $f(x) = |x^2 - 7x + 10|$. Find the sum of the local maximum and minimum values.

121. Let $f''(x) = 6x + 12, f'(-1) = 0$, and $f(1) = 12$. Find $f(-2)$.

122. The graph of $f(x)$ is shown. For what variables of x is $f(x)$ differentiable?



123. Bill Defense is fencing in a non-square rectangular area of 3,200 square feet. The cost of the fencing for two sides of the rectangle will cost $\$1.00$ per foot and the other two sides will cost $\$2.00$ per foot. What is the lowest possible cost for the fence?

124. Given: f is a continuous function on the interval $[0, 2]$ such that $\int_0^2 f(x)\mathrm{d}x = 5$. Find $\int_0^1 f(2y)\mathrm{d}y$.

Solutions

1. 2

2. C

3. -6

4. $(0, 1)$

5. 18

6. $10\frac{2}{3}$

7. -8.0

8. 193

9. 4890

10. 14.1

11. $(-4, 4)$

12. -2.24

13. $\sin x$

14. $2x + h - 8, h \neq 0$

15. 21.0

16. -1.5

17. 0.02 rad/s

18. 407.9 in^3

19. 114

20. 102.7

21. $4\sin(4x)$

22. x^2

23. 0.0000021

24. 1.6

25. $\frac{1}{2}$

26. 6.7

27. 36.4

28. Intermediate Value

29. 28,800 ft^2

30. 3.46 in/s

31. 3.8

32. 2.01

33. $\sin(x)$

34. x^2

35. 242

36. 8.00

37. 2.14

38. 22.6

39. -4.0

40. 85.3

41. 6434

42. 0.222

43. 5.9

44. 51,200

45. 7 in/s

46. 52.7

47. -0.27 m/s

48. -1.0 m/s^2

49. 4530

50. 642

51. 50.6

52. $-\frac{2^{-x}}{\ln 2}$

53. 5

54. $(1, 1)$

55. $1\frac{1}{3}$

56. Mean-value Theorem

57. E

58. $4\frac{1}{2}$

59. $2\frac{3}{4}$

60. 6

61. Intermediate Value Theorem

62. $22a - 18b$

63. 6

64. 1

65. $3x + 6y$

66. $20\frac{5}{6}$

67. -22

68. $(2, -1)$

69. $-2n(n^2 + 5)$

70. -.416

71. Rolle's Theorem

72. $1\frac{2}{3}$

73. $10\frac{2}{3}$

74. $-\frac{5}{12}$

75. $2x + 5y = 17$

76. -4

77. $y = -x$

78. $10\frac{2}{3}$

79. $x \cos x - \sin x$

80. 17

81. $-\frac{1}{16}$

82. $1\frac{1}{3}$

83. $222\frac{3}{4}$

84. -6

85. $-\frac{1}{8}\cos(4x)$

86. 7000 sq. ft.

87. $10\frac{2}{3}$

88. -4

89. $\frac{y-1}{1-x}$

90. 2

91. 1 & 2 but not 3

92. -11

93. 6.5

94. 2.5

95. 4.5 ft/sec

96. 2

97. 3 only

98. 102'0"

99. 4.25

100. -1

101. 126 cm^2/min

102. -2

103. 3

104. 1

105. 4.5

106. 8.9

107. 19.0

108. 13.6

109. 49.3

110. $\sin(2)$

111. 34.5

112. 55.11

113. 2385

114. 151°

115. I, III only

116. 2.09 cm/s

117. 9.63 cm

118. 258

119. $12x - 23$

120. $2\frac{1}{4}$

121. -6

122. $x < 1$ and $x > 1$

123. $\$320.00$

124. 2.5